

# Modeling of a Coaxial-Waveguide Power-Combining Structure

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**Abstract**—The modeling of the Kurokawa-Magalhaes power-combiner mounting structure is presented. The combiner is regarded as a four-port network with the ports created at the coaxial-waveguide apertures. By using a radial-modal wave approach, the admittance matrix for this network is determined. The theory is verified through an alternative approach based on the equivalence between a coaxial-waveguide junction and a dual-gap post structure.

## I. INTRODUCTION

THE CROSS-COUPLED coaxial-waveguide structure, introduced by Kurokawa and Magalhaes [1], has proved to be a successful module in the design of narrow-band power combiners [2]–[4]. Although this type of combiner has been in use for a long time, its design has been mostly empirical. A theoretical approach to the design of the coaxial-rectangular waveguide combiner has been initiated by Chang and Ebert [2]. They presented an equivalent circuit for the single coaxial-waveguide coupling structure and showed its application in the design of multiple diode combiners. The model presented in [2] neglects actual dimensions of the coaxial aperture and also does not take into account interactions between coaxial lines.

Recently, more accurate models of single coaxial-waveguide junctions have been presented by a number of other researchers [4], [6], [9], [15]. However, a solution to the multiple coaxial coupling structure is still missing.

One possible method to include multiple interactions in the combiner would be adaption of the modified approach by Joshi and Cornick [12]. In this method, coaxial-waveguide junctions would be replaced by equivalent strips with dual gaps [9], [10]. Although this approach is mathematically straightforward, problems do exist in using empirical factors to establish equivalence between round posts and flat strips, gaps, and coaxial entries [8]–[10].

The work presented here uses an alternative approach in which actual circular-rectangular geometry of the combiner is taken directly into account. The theory is based upon a radial-modal wave analysis, which is essentially equivalent to the method of images [4]–[6]. However, in contrast to the latter, the results are produced directly in terms of a fast converging series.

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## II. ANALYSIS OF THE COMBINER

The analyzed combiner module shown in Fig. 1 is created by two pairs of the coaxial-line-waveguide junctions. These junctions are located at the same transverse plane  $z = 0$ , with  $S_1$  and  $S_2$  the mean horizontal positions in the  $x$ -direction. It is assumed that the coaxial lines are identical with radii  $a$  and  $b$  and are located symmetrically:  $S_2 = A - S_1$ . Although the whole network includes six ports, it can be regarded as a four-port network with four ports created at the coaxial apertures, while the remaining two waveguide ports are terminated. In practice, the frequency of operation is such that only TEM waves can be propagated along the coaxial lines. In such situations, the network can be described in terms of voltages, currents, and admittances as seen by the TEM waves. The four-port can be characterized by the admittance matrix  $\{Y_{ik}\}$  with elements defined in [6]

$$Y_{ik} = \frac{J_i}{V_k} \quad \text{while } V_{i \neq k} = 0 \quad (1)$$

where  $V_k$  is a voltage applied at the  $k$ th coaxial aperture

$$V_k = - \int_a^b \bar{E}_{ak}(r) \cdot \bar{a}_r dr$$

where  $\bar{E}_{ak}$  is the electric field at the  $k$ th aperture, and  $J_i$  is a current at the  $i$ th aperture

$$J_i = \frac{2\Pi}{1n - a} \int_0^{2\Pi} \int_a^b \bar{H}_{ai}(r) \cdot \bar{a}_\phi dr d\phi$$

where  $\bar{H}_{ai}$  is the magnetic field at the  $i$ th aperture, and  $r, \phi, y$  are cylindrical coordinates.

The problem of determining  $Y_{ik}$  is stated as follows.

(i) Find the field in the waveguide due to the voltage  $V$  applied to one aperture while the remaining apertures are closed by perfect conductors.

(ii) Use definition (1) to calculate  $Y_{ik}$ .

As the structure of the combiner is symmetric, the matrix  $\{Y_{ik}\}$  is also symmetric and only elements  $Y_{ik}$  with  $k = 1$  need to be determined.

### A. Solution to the Field Problem

When voltage  $V$  is applied to the first aperture, it is equivalent to having a magnetic current  $\bar{M}$  on the aperture surface [6]. The current  $\bar{M}$  can be approximated by the

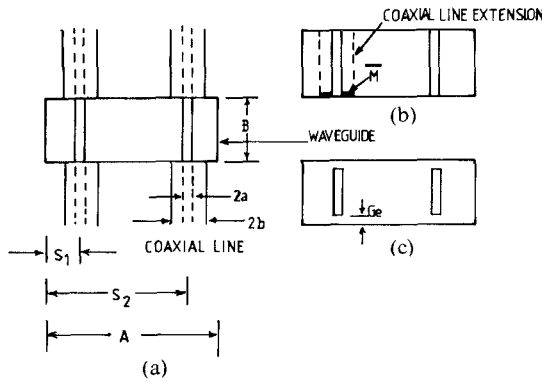


Fig. 1. Power-combiner configurations. (a) Cross section of the power-combiner module. (b) The combiner with the equivalent magnetic current  $\bar{M}$ . (c) The double-gap equivalent model.

TEM coaxial component [6], [14], [15], [17]

$$\bar{M} = \bar{a}_\phi \frac{V}{1n\left(\frac{b}{a}\right)} \frac{1}{r}. \quad (2)$$

For convenience, the field  $\bar{E}, \bar{H}$  produced by  $\bar{M}$  can be divided into two components. A suitable division can be achieved with the use of the Schelkunoff's field equivalence principle [15]. The procedure consists of two steps: 1) introduction of a conducting wall and determination of the field  $\bar{E}_1, \bar{H}_1$ ; 2) removal of the wall, introduction of the virtual source, and determination of the remaining field  $\bar{E}_2, \bar{H}_2$ .

In the problem considered here, the first component represents the field  $\bar{E}_1, \bar{H}_1$  produced by  $\bar{M}$  in the coaxial cavity created by extending the first outer coaxial conductor into the guide (see Fig. 1(b)). The field  $\bar{E}_1, \bar{H}_1$  exists only within the coaxial cavity. This field is radial TEM and corresponds to the currents  $I_c(y)$  and  $-I_c(y)$  flowing on the inner and outer coaxial conductors, respectively. From the analysis of the coaxial cavity, the current  $I_c$  is given by

$$I_c(y) = -j \frac{\cos k(y-B)}{Z_c \sin kB} \quad (3)$$

where

$$Z_c = \frac{Z_0}{2\Pi} 1n\left(\frac{b}{a}\right)$$

and is the characteristic impedance of the coaxial line,  $k$  is the wavenumber, and  $Z_0$  is the intrinsic impedance.

To recover the original field  $\bar{E}, \bar{H}$ , the outer cavity wall is removed and at its place a virtual source in the form of the current  $I_c(y)$  is introduced. The new current flows in the opposite direction to the current, which existed within the cavity. The virtual current produces the additional field  $\bar{E}_2, \bar{H}_2$ . The field  $\bar{E}_2, \bar{H}_2$  exists in the whole volume of the waveguide and generates currents  $I_1, I_2$  flowing on two cylindrical posts, which represent the inner conductors of the coaxial lines. As the posts are thin in comparison to the guide width, the currents can be considered circumferentially uniform.

Possible variation of the current densities can be assumed in the  $y$ -direction and the currents can be sought in the form of Fourier series

$$I_1(y) = \sum_{n=0}^{\infty} I_{1n} \frac{\epsilon_{0n}}{B} \cos k_{yn} y$$

$$I_2(y) = \sum_{n=0}^{\infty} I_{2n} \frac{\epsilon_{0n}}{B} \cos k_{yn} y. \quad (4a)$$

Also, the current  $I_c$  can be expanded into the Fourier series

$$I_c(y) = \sum_{n=0}^{\infty} I_{3n} \frac{\epsilon_{0n}}{B} \cos k_{yn} y \quad (4b)$$

where

$$I_{3n} = j \frac{k}{Z_c} \frac{1}{q_n^2}, \quad q_n^2 = k_{yn}^2 - k^2, \quad k_{yn} = \frac{n\Pi}{B}$$

and where  $\epsilon_{0n}$  is the Neuman factor.

The immediate problem is the determination of the coupling between the currents  $I_1, I_2, I_c$  and the field  $\bar{E}_2, \bar{H}_2$  in the waveguide. Convenient expressions for finding the required coupling are presented in the Appendix.

For purposes of analysis, only knowledge of the  $y$ -component of the electric field is required. The field  $\bar{E}_2$  can be regarded as a superposition of the individual fields produced by the currents  $I_1, I_2$ , and  $I_c$ . The  $y$ -component of the electric field due to each individual harmonic of the currents  $I_1, I_2$ , or  $I_c$  can be found by using formulas (A2) or (A10) given in the Appendix.

By including the fact that each harmonic of the  $y$ -component of the electric field  $\bar{E}_2$  is zero on the conducting surfaces of the posts, the following expressions for  $I_{1n}$  and  $I_{2n}$  coefficients in (4a) are obtained:

$$I_{2n} = - \frac{I_{1n} I_0(q_n a) + I_{3n} I_0(q_n b)}{K_0(q_n a) + C_n I_0(q_n a)} D_n$$

$$I_{1n} = - \frac{I_{2n} I_0(q_n a) D_n + I_{3n} (K_0(q_n b) + C_n I_0(q_n b))}{K_0(q_n a) + C_n I_0(q_n a)} \quad (5)$$

where  $I_0$  and  $K_0$  are modified Bessel and Hankel functions.

The expression (4a) in conjunction with (5) gives values of the currents flowing on the posts in the presence of the field  $\bar{E}_2, \bar{H}_2$ . The total current  $I_{1t}$  flowing on the inner conductor of the coaxial line in the presence of the field  $\bar{E}, \bar{H}$  is given as the sum of  $I_c$  and  $I_1$ . The total current  $I_{2t}$  on the second coaxial-line conductor is given by  $I_2$  only. This is so since the field  $\bar{E}, \bar{H}$  is identical with  $\bar{E}_2, \bar{H}_2$  for points in the vicinity of the second coaxial line.

The remaining part of the problem is the determination of the currents  $J_i$  required to find values of the admittance matrix.

#### B. Determination of the Admittance Matrix $\{Y_{ik}\}$

By using the radial-wave analysis presented in the Appendix, it can be shown that the currents  $J_i$  (1) can be

expressed by

$$J_i = \sum_{n=0}^{\infty} \frac{jk}{Z_c q_n^2} E_{yn}(S_i, r=b) \cos k_{yn} y_i + \begin{cases} \frac{-j \cos k(y_i - B)}{Z_c \sin kB} & \text{if } S_i = S_1 \\ 0 & \text{if } S_i = S_1 \end{cases} \quad (6)$$

where  $y_i = 0$  or  $B$ .

Expression (6) has been obtained with the use of definition (1). The first term in (6) represents a contribution of the TM radial magnetic field  $\bar{H}_2$  and the second term is due to TEM field  $\bar{H}_1$ . In the derivation of the first term, the property (A11) has been used. The final values of  $J_i$  and then  $Y_{ik}$  can be found by using the explicit expressions for the coefficients  $E_{yn}$  given in the Appendix.

The following equations present final results for the elements of the admittance matrix:

$$\begin{aligned} Y_{11} &= \begin{cases} -\frac{j \cot kB}{Z_c} \\ -\frac{j}{Z_c \sin kB} \end{cases} - \frac{Z_0}{2\pi Z_c} \sum_{n=0}^{\infty} \frac{\epsilon_{0n}}{B} [T_n Q_n + I_{2n} D_n I_0(q_n b) I_0(q_n a)] \cdot \begin{cases} 1 \\ (-1)^n \end{cases} \\ Y_{21} &= \begin{cases} -\frac{j \cot kB}{Z_c} \\ -\frac{j}{Z_c \sin kB} \end{cases} \\ Y_{31} &= -\frac{Z_0}{2\pi Z_c} \sum_{n=0}^{\infty} \frac{\epsilon_{0n}}{B} [D_n I_0(q_n b) T_n + I_{3n} I_0(q_n b) + I_{2n} I_0(q_n a) Q_n] \cdot \begin{cases} 1 \\ (-1)^n \end{cases} \\ Y_{41} &= \end{aligned} \quad (7)$$

where  $T_n = I_{1n} I_0(q_n a) + I_{3n} I_0(q_n b)$  and  $Q_n = K_0(q_n b) + C_n I_0(q_n b)$ . The coefficients  $C_n$  and  $D_n$  can be found in the Appendix.

### III. RESULTS

To confirm the validity of the analysis, a computer program for determining the admittance matrix of the power-combiner circuit has been developed. Cylindrical functions in expression (7) were calculated with the use of a polynomial approximation [16].

First, the analysis was applied to the single coaxial-waveguide junction, since it was easy to compare numerical results with some published experimental and theoretical data [6], [9], [10]. By setting coefficients  $D_n$  to zero, the value of  $Y_{11}$  gave the input admittance of the single coaxial-guide junction. Fig. 2 shows comparison between theoretical and experimental values [6], [10] of the input impedance  $Z = 1/Y_{11}$  for the standard X-band waveguide and a 50-Ω, 7-mm coaxial line. In calculations, only eight radial harmonics were required to produce results in error by less than 1 percent. The comparison shows very good agreement between theory and measurements.

In the next stage, the theory was verified for the case of the double coaxial-waveguide structure. As there was no experimental data available for the input admittances seen from the coaxial apertures, an indirect approach was used.

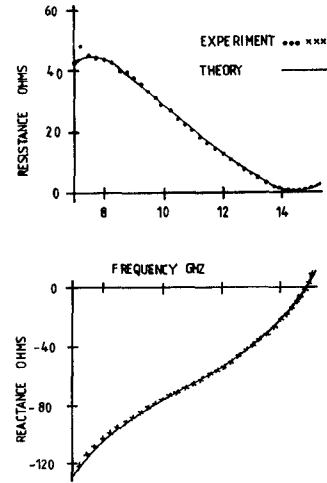


Fig. 2. Input impedance of the single coaxial-waveguide junction, comparison between experimental values and numerical values obtained from the radial-modal analysis. Dimensions: waveguide:  $A = 22.86$  mm,  $B = 10.16$  mm,  $S = A/2$ ; coaxial line:  $a = 1.52$  mm,  $b = 3.5$  mm.

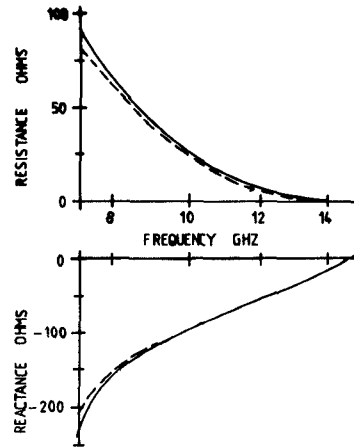


Fig. 3. Input impedance  $Z = 1/Y_{11}$  for the combiner, comparison between numerical results: radial-modal analysis ----; equivalent strips with gaps —; Dimensions: waveguide:  $A = 22.86$  mm,  $B = 10.16$  mm; coaxial lines:  $a = 1.52$  mm,  $b = 3.5$  mm,  $S_1 = 4.5$  mm; the equivalent gap height:  $G_e = 1.225$  mm.

A new computer program based on the multiple-post theory of [11] and [12] was developed. Round posts were modeled by strips and coaxial apertures by equivalent gaps [9]. The algorithm has been verified on the example of the single coaxial-guide junction and compared with the theoretical and experimental results presented in [9] and [10].

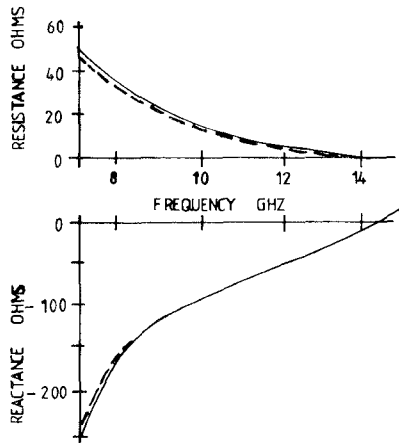


Fig. 4 Input impedance  $Z=1/Y_{11}$  for the combiner, comparison between numerical results: radial-modal analysis ----; equivalent strips with gaps —; Dimensions: waveguide:  $A=22.86$  mm,  $b=10.16$  mm; coaxial lines:  $a=1.52$  mm,  $b=3.5$  mm,  $S_1=7$  mm; the equivalent gap height:  $G_e=1.225$  mm.

The last program allowed for further verification of the radial-modal analysis.

Figs. 3 and 4 show the values of the input impedance  $Z=1/Y_{11}$  for the double coaxial-waveguide module calculated with use of the two methods based on equivalent strips and radial-modal analysis. Two extreme cases are presented: when the coaxial lines are adjacent to the waveguide walls (Fig. 3) and when the coaxial lines are close and centrally located in the waveguide (Fig. 4). In both cases, the waveguide was match terminated. Comparison shows that the results based on the two approaches are essentially the same. However, it should be noted that the algorithm based on the radial-modal approach was six times faster.

#### IV. CONCLUSIONS

A circuit model, given in terms of the elements of the admittance matrix, for the Kurokawa-Magalhaes power combiner has been found. The model has been verified by comparison with the modified multiple-post model of Joshi and Cornick [11]. It has been found that the accuracy of the radial-modal approach is similar to that obtained through the alternative approach with strips and gaps. However, the advantage of the new theory is that it does not require the use of any experimental factors. The calculations are also faster.

Although the analysis has been demonstrated for the special case of the double coaxial-waveguide module, it can be expanded for the multiple structure with arbitrary positions of the coaxial lines. The presented analysis opens the way for the accurate design of this type of power combiner at microwave and millimeter-wave frequencies.

#### APPENDIX

##### RADIATION OF AN ELECTRIC CURRENT FLOWING ON A CIRCULAR CYLINDER IN A RECTANGULAR GUIDE

In the analysis of the power combiner, there arises a need to determine the electromagnetic field on one cylindrical surface  $W_2$  produced by an electric current flowing

on another cylindrical surface  $W_1$ , both perpendicular to the broad wall of the guide. Two distinguished cases of interest are 1) when cylinders are noncoaxial, and 2) when cylinders are coaxial.

The problem stated in the above form is generally difficult to solve; therefore, some simplifying assumptions should be made. For purposes of analysis of the power combiner, the current is assumed to flow in the  $y$ -direction with the density uniform on the perimeter of  $W_1$ . Further considerations can be restricted to the current  $I(y)$  given by the single harmonic  $\cos k_{yn}y$ . A general solution can be obtained with the use of Fourier analysis. The wave produced by the current  $I(y)$  is radial and transverse magnetic (TM) to the  $y$ -direction. The wave can be considered axially symmetric for points in the vicinity of the two surfaces  $W_1$  and  $W_2$ . The last assumption is of an approximate nature. However, it should produce sufficiently accurate results if the diameters of the two cylinders are much smaller than the waveguide width.

It is worth noting that, as the field is TM radial, it is sufficient to obtain an expression for the  $y$ -component of the electric field. The remaining components can be produced from formulas held for TM waves [17].

##### Case 1)

The surfaces  $W_1$  and  $W_2$  are separated in space and given by the following expressions:

$$\begin{aligned} W_1: |r_1 - (S_1, z_1)| &= a_1, & 0 \leq y \leq B \\ W_2: |r_2 - (S_2, z_2)| &= a_2, & 0 \leq y \leq B \end{aligned} \quad (A1)$$

where  $S_i, z_i$  represent the mean positions in the  $x$ - and  $z$ -directions and  $a_i$  is the radius.

The wave generated by the current located on  $W_1$  takes the form of a radial wave traveling inwards to the surface  $W_2$ . From radial-wave theory [17], it is known that this type of wave is nonsingular at the origin of  $W_2$ . Therefore, it should be represented by a nonsingular Bessel function.

By including reciprocity between source and observation points, the following expression for the  $y$ -component of the electric field can be deduced [5], [6]:

$$E_{yn}(S_2, z_2, r_2) = \frac{jZ_0 q_n^2}{2\pi k} D_n(S_1, z_1, S_2, z_2) \cdot I_0(q_n a_1) I_0(q_n r_2) \cos k_{yn} y \quad (A2)$$

where

$$j = \sqrt{-1} \quad q_n^2 = k_{yn}^2 - k^2$$

and where  $k$  is the wavenumber,  $Z_0$  is the wave impedance, and  $I_0$  is the modified Bessel function. Expression (A2) is valid for  $r_2$  representing points close to the surface  $W_2$ . The coefficient  $D_n$  is unknown and can be determined by using modal analysis. When radii of  $W_1$  and  $W_2$  tend to zero,  $E_{yn}$  can be conveniently expressed in terms of the waveguide modes [8], [13] as follows:

$$E_{yn}(S_2, z_2, a_2 = 0) = \frac{jZ_0}{k} q_n^2 \sum_{m=1}^{\infty} \frac{\tau_{mn}(z_1, z_2)}{\Gamma_{mn}} \cdot \sin k_{xm} S_1 \sin k_{xm} S_2 \cos k_{yn} y$$

where

$$\Gamma_{mn}^2 = k_{xm}^2 + q_n^2, \quad k_{xm} = \frac{m\Pi}{A}. \quad (A3)$$

The function  $\tau_{mn}$  depends on the load conditions at the waveguide arms and has been given in [12] and [13]. For a case when the guide is match-terminated,  $\tau_{mn}$  reduces to the exponential factor

$$\tau_{mn}(z, z') = e^{-\Gamma_{mn}|z-z'|}.$$

By comparing (A2) and (A3),  $D_n$  can be determined and is given by

$$D_n(S_1, z_1, S_2, z_2) = 2\Pi \sum_{m=1}^{\infty} \frac{\tau_{mn}(z_1, z_2)}{\Gamma_{mn}} \sin k_{xm} S_1 \sin k_{xm} S_2. \quad (A4)$$

Convergence of the series in (A4) depends on the value of  $n$  and the distance  $|z_1 - z_2|$ . When  $|z_1 - z_2|$  is large, the series converges rapidly irrespective of the value of  $n$ . For  $|z_1 - z_2|$  small and  $n$  such that  $q_n^2 < 0$ , the convergence of the series in (A4) is slow. The convergence can be accelerated by adding and subtracting terms of the asymptotic series

$$F = 2\Pi \sum_{m=1}^{\infty} \frac{e^{-k_{xm}|z_2-z_1|}}{k_{xm}} \sin k_{xm} S_1 \sin k_{xm} S_2 \quad (A5)$$

which is represented analytically by the function

$$F = -\frac{A}{2} 1n \frac{\sinh \frac{\Pi}{2A} (|z_2 - z_1| - j(S_1 - S_2)) \sinh \frac{\Pi}{2A} (|z_2 - z_1| + j(S_1 - S_2))}{\sinh \frac{\Pi}{2A} (|z_2 - z_1| - j(S_1 + S_2)) \sinh \frac{\Pi}{2A} (|z_2 - z_1| + j(S_1 + S_2))} \quad (A6)$$

which, for  $z_1 = z_2 = 0$  and  $S_2 = A - S_1$ , can be reduced to

$$F = -\frac{A}{2} 1n \left( \cos \frac{\Pi S_1}{A} \right).$$

For  $n$  sufficiently large such that  $q_n^2 > 0$ , a more convenient representation of the coefficient  $D_n$  can be found. It should be noted that, for  $q_n^2 > 0$ , the current on the surface  $W_1$  produces a radial wave which strongly decays with distance. In this case, a rectangular guide can be regarded as a parallel-plate guide. If the surface  $W_1$  is close to the guide walls, single images of the source also have to be taken into account. Therefore, the coupling coefficient  $D_n$  can be represented by

$$D_n = K_0(q_n R) - \sum_i K_0(q_n R_i) \quad (A7)$$

where  $R = |(S_1, z_1) - (S_2, z_2)|$  and  $R_i$  means the distance between  $(S_2, z_2)$  and a possible image located at  $(S_i, z_i)$ .

For the purpose of the analysis of the power combiner, the  $y$ -component of the electric field due to the current  $I(y) = \cos k_{yn} y$  is given by (A2) in which  $D_n$  is repre-

sented by

$$D_n = \begin{cases} -1n \left( \cos \frac{\Pi S_1}{A} \right) - \frac{2\Pi}{A} \sum_{m=1}^{\infty} \left( \frac{\tau_{mn}}{\Gamma_{mn}} - \frac{1}{k_{xm}} \right) \\ \quad \cdot (-1)^m \sin^2 k_{xm} S_1, & \text{for } q_n^2 < 0 \\ K_0(q_n R) - \sum_i K_0(q_n R_i), & \text{for } q_n^2 > 0 \end{cases} \quad (A8)$$

Case 2)

The cylindrical surfaces  $W_1$  and  $W_2$  are coaxial and described by the following expressions:

$$\begin{aligned} W_1: & \quad |\bar{r}_1 - (S_1, z_1)| = a_1 \\ W_2: & \quad |\bar{r}_2 - (S_1, z_1)| = a_2 \end{aligned} \quad 0 \leq y \leq B. \quad (A9)$$

The radial waves approaching  $W_1$  travel inwards and outwards to  $W_2$ . The expression representing the  $y$ -component of the electric field should include a singularity at the source [17]. A suitable form for the  $y$ -component of the electric field due the current  $I(y) = \cos k_{yn} y$  is given as follows [13]:

$$E_{yn}(S_2, z_2, r_2) = \frac{jZ_0}{2\Pi k} q_n^2 \left\{ \begin{aligned} & K_0(q_n r_2) I_0(q_n a_2) \\ & K_0(q_n a_2) I_0(q_n r_2) \\ & + C_n(S_2, z_2) I_0(q_n r_2) I_0(q_n a_2) \end{aligned} \right\} \cos k_{yn} y \quad (A10)$$

where the upper row holds for  $r_2 > a_2$  and lower row holds

for  $r_2 < a_2$ . The coupling coefficient  $C_n$  represents the interaction of the source with the waveguide walls and is given by

$$C_n = \begin{cases} 1n(\beta q_n) + 1n \left( \frac{\sin \frac{\Pi S_1}{A}}{\frac{\Pi}{2A}} \right) \\ \quad + \frac{2\Pi}{A} \sum_{m=1}^{\infty} \sin^2 k_{xm} S_1 \left( \frac{\tau_{mn}}{\Gamma_{mn}} - \frac{1}{k_{xm}} \right) \\ \quad \text{for } q_n^2 < 0 \\ - \sum_i K_0(q_n R_i), & \text{for } q_n^2 > 0 \end{cases}$$

where  $1n\beta = -0.1159$ ,  $R_i$  is the distance between the source and its image, and  $S_1 = A - S_2$ . The other components of the electromagnetic field in cases 1) and 2) can be obtained from the formulas holding for the TM radial harmonics [17]. The component  $\phi$  of the magnetic field required in (1) is given as an infinite series of spatial

harmonics. Each harmonic can be determined by using the following relationship [17]:

$$H_{\phi n}(S_i, z_i, r) = \frac{jk}{Z_0 q_n^2} \frac{\partial E_{yn}}{\partial r}. \quad (A11)$$

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